# Markovian Queueing Model with Single Working Vacation and Server Breakdown

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In this article, we investigate a Markovian queuing model with server breakdown and Single working vacation. Arrival follows Poisson process with parameter $\lambda$. Service while the single working vacation epoch, normal working epoch together with vacation epoch are all exponentially assigned with rate $\mu_b$, $\mu_v$ and $\eta$respectively. After taking first vacation the server wait idle in the system to serve. This type of vacation is called Single Working Vacation (SWV). If the queue length increases, service rate changes from slow rate to normal rate. When the server may subject to sudden breakdown with rate $\alpha$ and after it should be repaired and goes to normal service with rate $\beta$. This queue model has been analysed with the help of Matrix Geometric Method (MGM) to find steady state probability vectors. Using it some performance measure is also determined.

***AMS subject classification number***— 60K25,60K30 and 90B22
***Key Words*** —- Working Vacation (WV), Single Working Vacation (SWV) , Stability condition, Server breakdown, Matrix Geometric Method (MGM);

## 1 Introduction

Queueing systems with SWV and absolute service have acquired importance over the most recent twenty years because of large extent uses mainly manufacturing system, service system, telecommunications, and computer system. Its discoveries might be utilized to give quicker client support, increment traffic stream, further develop request shipments from a stockroom, or plan information organizations and call focuses. Numerous significant utilization of the queueing theory are traffic flow vehicles, airplane, individuals, correspondences, booking patients in medical clinics, occupations on machines, programs on PC, and office configuration banks, mail depots, stores. In numerous genuine queueing circumstances, after assistance fulfillment, if no customer in the queue, a server goes on a vacation epoch. This type is known as vacation queue. Using survey paper by Doshi (1986) many researchers introduced queueing model with vacations.

Servi and Finn (2002) developed an M/M/1 queueing model upon WV. Wu and Takagi (2003) analysed an M/G/1 queueing model upon MWV. Analysis of GI/M/1 queueing model upon MWV studied by Baba (2005). Server will only take one WV if the queue become null. So, if the queue has no customers when the server come back from SWV, he will idle on the system and wait for the customers to arrive instead of picking up another WV. A multi-server system with an SWV were proposed by Lin and Ke (2009). The use of inactive epoch a M/G/1 model were studied by Levy and Yechiali (1975).

The vacation models and the model in which the server may goes to breakdowns and repairs, is well ascertained in survey papers B.T. Doshi. William J. Gray et al (2000) overworked on multiple types of server breakdowns in queueing theory. A queueing model with server breakdowns, repairs, vacations, and backup server is studied by Srinivas R.Chakravarthy (2020).Agelenbe et al (1991) analyzed the queues with negative arrivals. A matrix-Geometric method approach is a useful tool for solving the more complex queueing problems. Neuts (1978) deliberate Markov c hains with applications queueing theory, which have a matrix geometric invariant probability vector. Neuts (1981) derived Matrix Geometric solution in stochastic models.

Transient solution for the queue-size distribution in a finite-buffer model with general independent input stream and single working vacation was explained by Wojciech M Kempa et al (2018). Rachita Sethi et al (2019) studied performance analysis of machine repair problem with working vacation and service interruptions. Seenivasan et al (2021) studied performance examination of two heterogeneous servers queuing model with an irregularly reachable server utilizing MGM. Seenivasan et al (2021) investigated a retrial queueing model with two heterogeneous servers using the MGM. Praveen Deora et al (2021) analyzed the cost analysis and optimization of machine repair model with working vacation and feedback policy. M/M/1 queueing model with working vacation and two type of server breakdown was discussed by Praveen Kumar Agrawal et al (2021).

[1] Our study, deals with an SWV and server breakdown in M/M/1 queueing model. In accordance with FCFS principle customers are served. This model has been analysed using MGM. The excess of this study designated as follows. We providing construction of model in section 2. Performance measures formalized in section 3. Mathematical illustrations solved in section 4. And brief conclusion in final section.

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## 2 Construction of the Model

We consider a Single Working Vacation (SWV) and Server breakdown in M/M/1 model. The customers show up in line as per Poisson process with parameter $\lambda$. They create a queue dependent on her/his request for appearance. At a normal working period, influx customers served at a service rate $\mu_b$, following an exponential distribution. The server starts a single vacation of arbitrary length if there is no customer in the system with parameter $\eta$ follows an exponential distribution. During a SWV period, influx customers get service with rate $\mu_v$, following an exponential distribution. If the queue forms, then the server chop and shift its rate from $\mu_v$ to $\mu_b$, the normal working interval starts. For next vacation server waits idle to serve influx customers. This type of vacation is called Single Working Vacation (SWV). If not, the server starts a normal working period when a customer arrival occurs.

When the server may subject to sudden breakdown with rate $\alpha$ and after it should be repaired and goes to normal service with rate $\beta$. The transition rate diagram is shown in Figure 1.

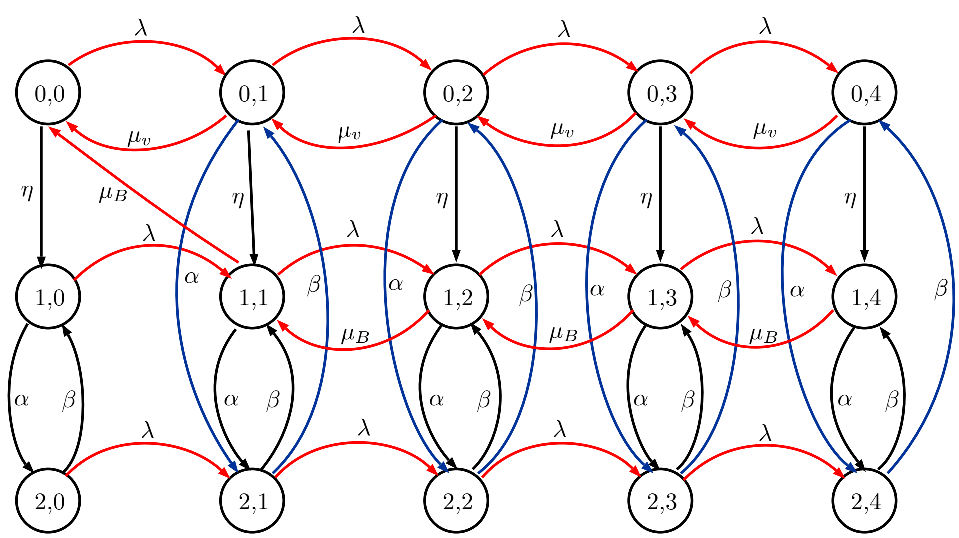


Figure 1: The transition rate diagram

Let $\{k(t), n(t) : t \geq 0\}; \lim_{t\to\infty} p\{k(t) = i, n(t) = j\}$ be a Markov process, where $k(t)$ and $n(t)$ represent state of process at time t respectively.
$k(t) = 0$, when server is on SWV,
$k(t) = 1$, when server is on normal working epoch
$k(t) = 2$, when server is on breakdown
n(t) denotes total customer in the system.
The Quasi-birth and death Process along with the state space $\Omega$ as follow

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$\Omega = \{(0,0)U(1,0)U(2,0)U(i,j); i = 0,1,2, j = 1,2,....,n \geq 1\}$

Consider a QBD process with Infinitesimal generator matrix Q is presented below

$$Q = \begin{pmatrix} A_0 & C_1 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ C_0 & C_2 & C_3 & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & C_3 & C_2 & C_3 & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & C_3 & C_2 & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & \cdots & C_3 & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{pmatrix}$$

Where

$$A_0 = \begin{pmatrix} -(\lambda+\eta) & \eta & 0 \\ 0 & -(\lambda+\alpha) & \alpha \\ 0 & \beta & -(\beta+\lambda) \end{pmatrix}; C_1 = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix};$$

$$C_0 = \begin{pmatrix} \mu_v & 0 & 0 \\ \mu_b & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; C_2 = \begin{pmatrix} -(\lambda+\mu_v+\alpha+\eta) & \eta & \alpha \\ 0 & -(\lambda+\mu_b+\alpha) & \alpha \\ \beta & \beta & -(2\beta+\lambda) \end{pmatrix};$$

$$C_3 = \begin{pmatrix} \mu_v & 0 & 0 \\ 0 & \mu_b & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

We define $p_{ij} = \{k = i, n = j\}$; where j denote number of customers in the system & i denote the server state.

Probability vector are defined as $P = (P_0, P_1, P_2, \ldots)$ where, $P_j = (p_{0j}, p_{1j}, p_{2j})$, $j = 0,1,2,3,...$

The static probability row matrix is represented by using $PQ = O$. (1)

$P_j = P_0 R^j$ where $j \geq 1$ (2)

The normalizing equation is defined by

$P_0 [I-R]^{-1} e = 1$ (3)

Where 'e' is the column unit vector with all its element equal to one.

The static condition of such a QBD, (See Neuts (1981)) can be obtained by the drift condition

$PC_1e < PC_3e$ (4)

Where the row vector $P = (P_0, P_1, P_2)$ is obtained from the Infinitesimal generator

$S = C_1 + C_2 + C_3$. S is given by

$$S = \begin{pmatrix} -(\alpha+\eta) & \eta & \alpha \\ 0 & -\alpha & \alpha \\ \beta & \beta & 2\beta \end{pmatrix} \quad (5)$$

$S$ is irreducible and the row vector $P$ can be shown to be unique such that

$PS = 0$ and $Pe = 1$ (6)

From Equation(6), we have

$P_1 = (\frac{2\eta-\alpha}{\alpha})P_0$

$P_2 = (\frac{\eta+\alpha}{\beta})P_0$

$P_0 = [1 + (\frac{\eta+\alpha}{\beta}) + (\frac{2\eta-\alpha}{\alpha})]^{-1}$ (7)

The static condition takes format

$\lambda[P_0 + P_1 + P_2] < \mu_b P_0 + \mu_v P_1$ (8)

Equation(5) gives the static probability of $S$.

Once the rate matrix $R$ obtained, the probability vectors $P_j$'s ($j \geq 1$) are obtained from Eq.(2) and Eq.(3).

# 3 Performance Measures

Performance measure have been found using steady-state probabilities as given below
When the server is idle mean no. of customers presented $E(I) = P_0$ (9)
When the server is SWV mean no. of customers presented
$$E(SWV) = \sum_{j=0}^{\infty} jp_{0j} \tag{10}$$
When the server is normal busy period mean no. of customers presented
$$E(BP) = \sum_{j=1}^{\infty} jp_{1j} \tag{11}$$
When the server is on breakdown mean no. of customers presented
$$E(BD) = \sum_{j=1}^{\infty} jp_{1j} \tag{12}$$
Mean no. of customer in the system is
$$E(N) = E(I) + E(WV) + E(BP) + E(BD) \tag{13}$$

# 4 Mathematical Study

Here, we make mathematical calculation for model given by the segment above. Our goals are to show effect of a parameter on system features. By modifying $\lambda$, four illustrations are presented in these sections.
The parameter $\lambda$ value varies and all other argument values are fixed. Illustration 1 to Illustration 4 is presented below.

**Illustration 1**

We take $\lambda = 0.1$,$\mu_b = 0.6$,$\mu_v = 0.5$,$\alpha = 0.2$,$\eta = 0.5$,$\beta = 0.7$ and the rate matrix is

$$R = \begin{pmatrix} 0.0942 & 0.0781 & 0.0193 \\ 0.0126 & 0.1347 & 0.0167 \\ 0.0534 & 0.1106 & 0.0834 \end{pmatrix}$$

**Table 1**.Probability vectors

| | $p_{0j}$ | $p_{1j}$ | $p_{2j}$ | Total |
|---|---|---|---|---|
| $P_0$ | 0.1215 | 0.5476 | 0.1479 | 0.8170 |
| $P_1$ | 0.0262 | 0.0996 | 0.0232 | 0.1496 |
| $P_2$ | 0.0050 | 0.0181 | 0.0042 | 0.0273 |
| $P_3$ | 0.0009 | 0.0033 | 0.0007 | 0.0049 |
| $P_4$ | 0.0002 | 0.0006 | 0.0001 | 0.0009 |
| $P_5$ | 0.0000 | 0.0001 | 0.0000 | 0.0001 |
| Total | | | | 0.9998 |

By substituting $R$ matrix in equation (1) vector $P_0$ are obtained and normalization equation $P_0\left[I-R\right]^{-1}e = 1$ for the mathematical argument selected previously, row vector $P_1$ is granted by $P_0 = (0.1215, 0.05476, 0.1479)$. More, the balance vector $P_j$'s gained from $P_j = P_0R^j, j = 1, 2, 3, \ldots$ and are shown in Table 1. Column 2, 3 and 4 contains the three elements of $P_j, j = 0, 1, 2, \ldots$. Final column constitutes the total of two elements. Total probability was confirmed to be $0.9998 \approx 1$.

**Illustration 2**

We take $\lambda = 0.2$,$\mu_b = 0.6$,$\mu_v = 0.5$,$\alpha = 0.2$,$\eta = 0.5$,$\beta = 0.7$ and the rate matrix is

$$R = \begin{pmatrix} 0.1807 & 0.1644 & 0.0313 \\ 0.0250 & 0.2694 & 0.0274 \\ 0.1024 & 0.2328 & 0.1507 \end{pmatrix}$$

**Table 2**.Probability vectors

| | $p_{0j}$ | $p_{1j}$ | $p_{2j}$ | Total |
|---|---|---|---|---|
| $P_0$ | 0.1655 | 0.3782 | 0.0979 | 0.6416 |
| $P_1$ | 0.0494 | 0.1519 | 0.0303 | 0.2316 |
| $P_2$ | 0.0158 | 0.0561 | 0.0103 | 0.0822 |
| $P_3$ | 0.0053 | 0.0201 | 0.0036 | 0.0290 |
| $P_4$ | 0.0018 | 0.0070 | 0.0013 | 0.0101 |
| $P_5$ | 0.0006 | 0.0025 | 0.0004 | 0.0035 |
| $P_6$ | 0.0002 | 0.0009 | 0.0002 | 0.0013 |
| $P_7$ | 0.0001 | 0.0003 | 0.0001 | 0.0005 |
| $P_8$ | 0.0000 | 0.0001 | 0.0000 | 0.0001 |
| Total | | | | 0.9999 |

By substituting $R$ matrix in equation (1) vector $P_0$ are obtained and normalization equation $P_0\left[I-R\right]^{-1}e = 1$ for the mathematical argument selected previously, row vector $P_1$ is granted by $P_0 = (0.1655, 0.3782, 0.0979)$. More, the balance vector $P_j$'s gained from $P_j = P_0 R^j, j = 1, 2, 3, \ldots$ and are shown in Table 1. Column 2, 3 and 4 contains the three elements of $P_j, j = 0, 1, 2, \ldots$. Final column constitutes the total of two elements. Total probability was confirmed to be $0.9999 \approx 1$.

**Illustration 3**

We take $\lambda = 0.3$,$\mu_b = 0.6$,$\mu_v = 0.5$,$\alpha = 0.2$,$\eta = 0.5$,$\beta = 0.7$ and the rate matrix is

$$R = \begin{pmatrix} 0.2587 & 0.2552 & 0.0391 \\ 0.0363 & 0.4018 & 0.0345 \\ 0.1452 & 0.3612 & 0.2067 \end{pmatrix}$$

**Table 3**.Probability vectors

| | $p_{0j}$ | $p_{1j}$ | $p_{2j}$ | Total |
|---|---|---|---|---|
| $P_0$ | 0.1615 | 0.2511 | 0.0640 | 0.4766 |
| $P_1$ | 0.0602 | 0.1612 | 0.0282 | 0.2536 |
| $P_2$ | 0.0257 | 0.0919 | 0.0139 | 0.1315 |
| $P_3$ | 0.0120 | 0.0485 | 0.0070 | 0.0675 |
| $P_4$ | 0.0059 | 0.0251 | 0.0036 | 0.0346 |
| $P_5$ | 0.0030 | 0.0129 | 0.0018 | 0.0177 |
| $P_6$ | 0.0015 | 0.0066 | 0.0009 | 0.0090 |
| $P_7$ | 0.0008 | 0.0034 | 0.0005 | 0.0047 |
| $P_8$ | 0.0004 | 0.0017 | 0.0002 | 0.0023 |
| $P_9$ | 0.0002 | 0.0009 | 0.0001 | 0.0012 |
| $P_{10}$ | 0.0001 | 0.0005 | 0.0001 | 0.0007 |
| $P_{11}$ | 0.0001 | 0.0002 | 0.0000 | 0.0003 |
| $P_{12}$ | 0.0000 | 0.0001 | 0.0000 | 0.0001 |
| Total | | | | 0.9997 |

By substituting $R$ matrix in equation (1) vector $P_0$ are obtained and normalization equation $P_0 [I - R]^{-1} e = 1$ for the mathematical argument selected previously, row vector $P_1$ is granted by $P_0 = (0.1615, 0.2511, 0.0640)$. More, the balance vector $P_j$'s gained from $P_j = P_0 R^j, j = 1, 2, 3, \ldots$ and are shown in Table 1. Column 2, 3 and 4 contains the three elements of $P_j, j = 0, 1, 2, \ldots$. Final column constitutes the total of two elements. Total probability was confirmed to be $0.9997 \approx 1$.

**Illustration 4**

We take $\lambda = 0.4$,$\mu_b = 0.6$,$\mu_v = 0.5$,$\alpha = 0.2$,$\eta = 0.5$,$\beta = 0.7$ and the rate matrix is

$$R = \begin{pmatrix} 0.3275 & 0.3420 & 0.0441 \\ 0.0453 & 0.5253 & 0.0392 \\ 0.1802 & 0.4836 & 0.2546 \end{pmatrix}$$

**Table 4**.Probability vectors

| | $p_{0j}$ | $p_{1j}$ | $p_{2j}$ | Total |
|---|---|---|---|---|
| $P_0$ | 0.1304 | 0.1578 | 0.0421 | 0.3303 |
| $P_1$ | 0.0574 | 0.1478 | 0.0227 | 0.2279 |
| $P_2$ | 0.0296 | 0.1083 | 0.0141 | 0.1520 |
| $P_3$ | 0.0171 | 0.0738 | 0.0091 | 0.1000 |
| $P_4$ | 0.0106 | 0.0491 | 0.0060 | 0.0657 |
| $P_5$ | 0.0068 | 0.0323 | 0.0039 | 0.0430 |
| $P_6$ | 0.0044 | 0.0212 | 0.0026 | 0.0282 |
| $P_7$ | 0.0029 | 0.0139 | 0.0017 | 0.0185 |
| $P_8$ | 0.0019 | 0.0091 | 0.0011 | 0.0121 |
| $P_9$ | 0.0012 | 0.0059 | 0.0007 | 0.0078 |
| $P_{10}$ | 0.0008 | 0.0039 | 0.0005 | 0.0052 |
| $P_{11}$ | 0.0005 | 0.0025 | 0.0003 | 0.0033 |
| $P_{12}$ | 0.0003 | 0.0017 | 0.0002 | 0.0022 |
| $P_{13}$ | 0.0003 | 0.0011 | 0.0001 | 0.0014 |
| $P_{14}$ | 0.0001 | 0.0007 | 0.0001 | 0.0009 |
| $P_{15}$ | 0.0001 | 0.0004 | 0.0001 | 0.0006 |
| $P_{16}$ | 0.0001 | 0.0003 | 0.0000 | 0.0004 |
| $P_{17}$ | 0.0000 | 0.0002 | 0.0000 | 0.0002 |
| $P_{18}$ | 0.0000 | 0.0001 | 0.0000 | 0.0001 |
| Total | | | | 0.9998 |

By substituting $R$ matrix in equation (1) vector $P_0$ are obtained and normalization equation $P_0\left[I-R\right]^{-1}e = 1$ for the mathematical argument selected previously, row vector $P_1$ is granted by $P_0 = (0.1304, 0.1578, 0.0421)$. More, the balance vector $P_j$'s gained from $P_j = P_0R^j, j = 1, 2, 3, \ldots$ and are shown in Table 1. Column 2, 3 and 4 contains the three elements of $P_j, j = 0, 1, 2, \ldots$. Final column constitutes the total of two elements. Total probability was confirmed to be $0.9998 \approx 1$.

**Table 4**.Performance Measures

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| $\lambda$ | $E(I)$ | $E(SWV)$ | $E(BP)$ | $E(BD)$ | $E(N)$ |
|---|---|---|---|---|---|
| 0.1 | 0.8170 | 0.0397 | 0.1486 | 0.0347 | 1.0400 |
| 0.2 | 0.6416 | 0.1090 | 0.3736 | 0.0708 | 1.1950 |
| 0.3 | 0.4766 | 0.2079 | 0.7529 | 0.1128 | 1.5502 |
| 0.4 | 0.3303 | 0.3412 | 1.4211 | 0.1792 | 2.2718 |

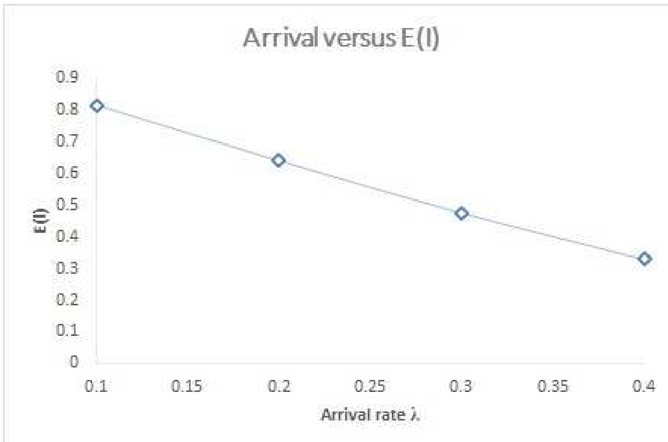


**Figure 2**

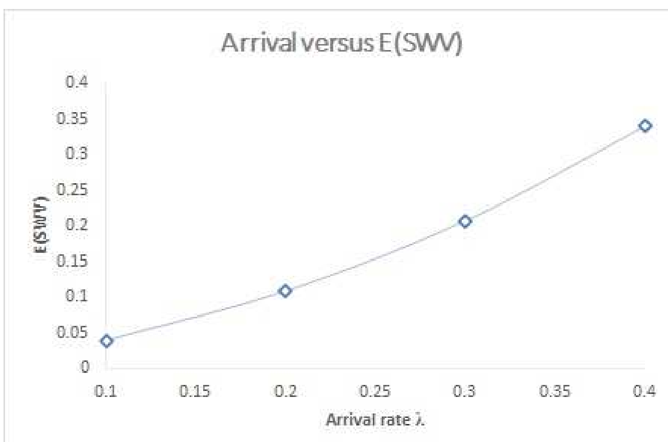


**Figure 3**

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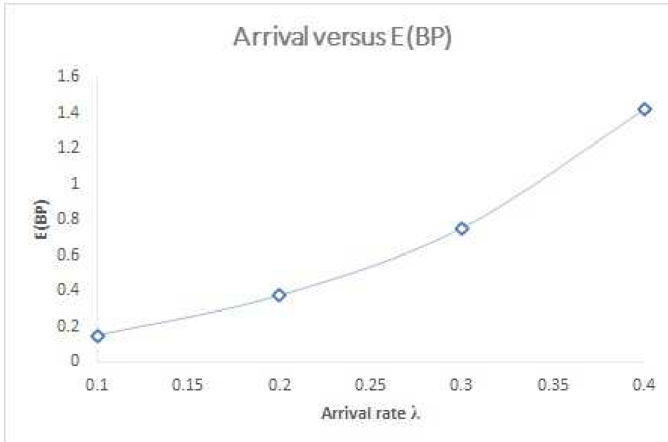


**Figure 4**

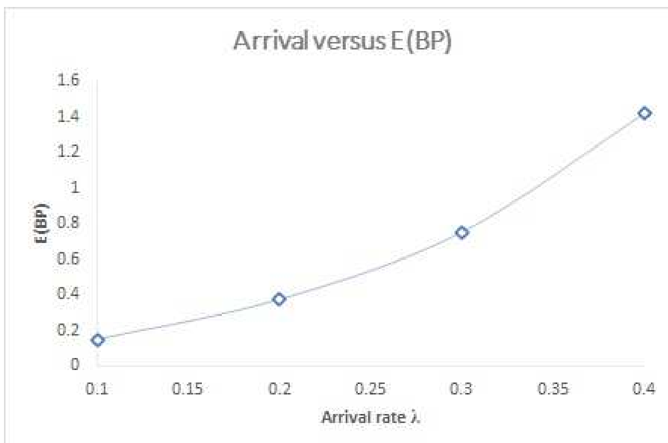


**Figure 5**

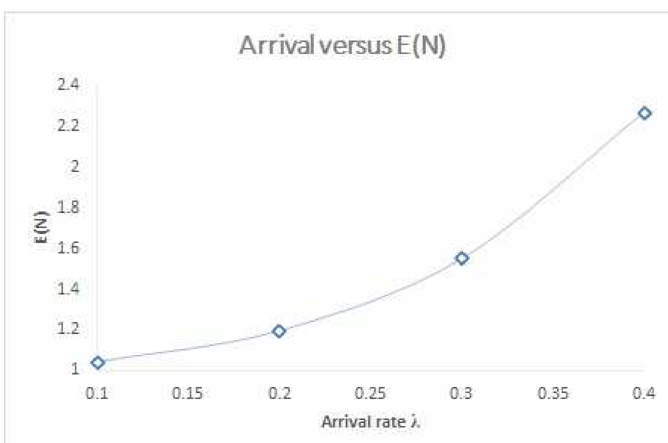

**Figure 6**

Out of the above figures, we derived few performance measurements with the effect of $\lambda$ such as mean no., of customer if server is idle, mean no., of customer if server is on SWV, mean no., of customer if server is on busy period, mean no., of customer if server is on breakdown and mean no., of customers throughout system respectively. From Figure 2 shows that arrival rate increases, mean no., of customer if server is idle decreases, Figure 3, Figure 4, Figure 5 and Figure 6 shows arrival rate increases, mean no., of customer if server is SWV, busy period, breakdown and mean no., of customers throughout system increases.

# 5 CONCLUSION

In this article, we have studied a single-server queueing model along with SWV and server breakdown. We derived the static probability row vector by MGM and also we derived some performance measures for mean no., of customers in the system during server is idle, SWV, normal busy period, breakdown and mean no., of customers throughout system respectively with the effect of $\lambda$.

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