Numerical analysis of Non-Linear Waves Propagation and interactions in Plasma

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Abstract

Solitary wave propagation and interaction in plasma using numerical tools like Galerkin Finite Element scheme are discussed in this paper. A one-dimensional nonlinear Schrodinger-Korteweg De-Vries (Sch-KdV) equation is taken as model equation for Non-linear waves propagation in the said media. The derived system, with the help of cubic B-spline source functions are engaged as element and weight functions, after finite element formulation is solved with Runge Kutta Fourth Order method (RK^4). Previously the finite element methods with some numerical simulations do not exhibit the complex nature of wave interaction, especially solitary wave interaction. A combination of Galerkin Finite Element scheme with RK^4 is a very prominent instrument to study the nature of Non-linear evolution equations in ionic medias, which is the novelty of the paper.

Key words: Schrödinger - Korteweg - De Vries (Sch-KdV)equations, Galerkin Finite Element Scheme,Cubic B-spline source functions, Solitary Wave

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1 Introduction

Several physical phenomena are described either by nonlinear coupled partial differential equations or by nonlinear evolution equation. This Non-linear wave propagation phenomenon appears in one or other ways can be well explained by travelling and solitary wave solution of the said equations. Most of these equations do not have an analytical solution, or it is extremely difficult and expensive to compute their analytical solutions. Hence numerical study of these nonlinear partial differential equations is important in practice. The Non-linear

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waves propagation in plasma can also be explained by these solutions. In the past study, many methods for finding the Solitary and periodic solutions [1]-[8] and numerical method [8]-[12],[21]-[24] are used for Non-linear evolution equations (NLEEs).
In this paper, we study a Galerkin finite element Scheme for the 1D nonlinear Schrödinger -Korteweg-De -Vries (Sch-KdV) equation by using linear finite elements in space and extrapolation to remove the nonlinear term. We discuss the properties of this method and compare its accuracy with previous studies. The interaction of two solitons is also studied. Moreover, the propagation of the Maxwellian initial condition is simulated.
The outline of this paper is as follows, In the next section the model equation is discretized to form a numerical scheme. In section 3 a numerical scheme is developed and results are explained graphically. Finally, we give a brief conclusion in Section 4

2 Model Equation and Discretization

Non-linear waves propagation and interactions in plasma for this purpose we consider the 1D nonlinear Schrödinger -Korteweg-De -Vries (Sch-KdV) equation [13]-[15] as model equation as -

$$i\theta_t = \theta_{xx} + \theta\upsilon \tag{2.1}$$

$$\upsilon_t = -6\theta\upsilon_x - \upsilon_{xxx} + (|\theta|^2)_x \tag{2.2}$$

Here $\theta(x,t)$is complex function and υ(x,t)is real-valued function. This system appeared as model equation for describing various types of wave propagation such as Langmuir wave, dust-acoustic wave and electromagnetic waves in plasma physics. with initial conditions

$$\theta(x,0) = f(x) = 9\sqrt{2}\, e^{i\alpha x}\, k^2 \text{sech}^2(kx) \tag{2.3}$$

$$v(x,0) = g(x) = \frac{\alpha + 16k^2}{3} - 6k^2 \tanh^2(kx) \tag{2.4}$$

and boundary conditions

$$\theta(t,a) = 0,\ v(t,b) = 0,\ \ x \in [a,b]\, and\ t \in [0,T] \tag{2.5}$$

Here $\theta = \theta(x,t)$ and $\upsilon = \upsilon$(x,t) are going to be considered as sufficiently differentiable functions.
We multiplied weight function to the equations (2.1)-(2.2) and integrated over the x domain for finite element method [16]-[20], so we get

$$\int_a^b (i\omega\theta_t - \omega\theta_{xx} - \omega\theta\upsilon)dx = 0 \tag{2.6}$$

$$\int_a^b (\omega\upsilon_t + 6\omega\theta\upsilon_x + \omega\upsilon_{xxx} - \omega(|\theta|^2)_x)dx = 0 \tag{2.7}$$

The domain [a, b] of x is separated into N finite subdivision as

2

$$a = x_0 < x_1 < x_2 < ... < x_{N-1} < x_N = b$$

Here nodal point is $\{x_m\}_{m=0}^{N}$ i.e. m = 0,1,2,...,N and length of subdivision will be $h = x_{m+1}$ - x_m . We construct the approximate solutions for the system with cubic B-spline base functions

$$\theta_N(x,t) = \sum_{j=-1}^{N+1} \psi_j(x) u_j(t) \tag{2.8}$$

$$\upsilon_N(x,t) = \sum_{j=-1}^{N+1} \psi_j(x) v_j(t) \tag{2.9}$$

where u_j(t) and v_j(t) are function of time t and ψ_j(x) are function of x, called element size functions. A local coordinate $\xi = x - x_m$ for 0$\leq \xi \leq$ h introduced for cubic B-spline base functions with typical element $[x_m, x_{m+1}]$, which has the form;

$$\begin{aligned}
\psi_{m-1} &= \frac{(h-\xi)^3}{h^3} \\
\psi_m &= \frac{(h^3 + 3h^2(h-\xi) + 3h(h-\xi)^2 - 3(h-\xi)^3)}{h^3} \\
\psi_{m+1} &= \frac{(h^3 + 3h^2\xi + 3h\xi^2 - 3\xi^3)}{h^3} \\
\psi_{m+2} &= \frac{\xi^3}{h^3}
\end{aligned} \tag{2.10}$$

The approximate solutions of Eqs.((2.8)-(2.9)) with element size function eq.(2.10) may be define as with typical element $[x_m, x_{m+1}]$;

$$\theta_N(\xi,t) = \sum_{j=m-1}^{m+2} u_j^e(t) \psi_j^e(\xi) \tag{2.11}$$

$$\upsilon_N(\xi,t) = \sum_{j=m-1}^{m+2} v_j^e(t) \psi_j^e(\xi) \tag{2.12}$$

The point-wise values of θ_N and υ_N in terms u and v will be

$$\theta_N(x_m,t) = u_{m-1} + 4u_m + u_{m+1} \tag{2.13}$$

$$\upsilon_N(x_m,t) = v_{m-1} + 4v_m + v_{m+1} \tag{2.14}$$

So Eqs. ((2.6)-(2.7)) with $[x_m, x_{m+1}]$ will be

$$\int_{x_m}^{x_{m+1}} (i\omega\theta_t - \omega\theta_{xx} - \omega\theta\upsilon)dx \tag{2.15}$$

$$\int_{x_m}^{x_{m+1}} (\omega\upsilon_t + 6\omega\theta\upsilon_x + \omega_{xx}\upsilon_x - 2\omega\theta\theta_x)dx + [\omega\upsilon_{xx} - \omega_x\upsilon_x] \tag{2.16}$$

here weight function ω_i with size functions ψ_j are takenfor the Galerkin finite element method, Substituting Eqs. ((2.11)-(2.12)) into Eqs. ((2.15)-(2.16)),we get

$$\sum_{j=m-1}^{m+2}\{\int_0^h[(i\psi_i\psi_j)\dot{u}_j-(\psi_i\psi_j^{''})u_j-\sum_{k=m-1}^{m+2}((\psi_i\psi_j\psi_k)u_j)u_k]dx\}=0 \tag{2.17}$$

$$\sum_{j=m-1}^{m+2}\{\int_0^h[(\psi_i\psi_j)\dot{v}_j+(\psi_j^{"}\psi_k')v_j+\sum_{k=m-1}^{m+2}((6(\psi_i\psi_j\psi_k')u_j)v_k-2((\psi_i\psi_j \psi_k')u_j)u_k)]dx+[((\psi_i\psi_j^{"})-(\psi_i'\psi_j'))v_j]_0^h\}=0 \tag{2.18}$$

where i, j, k = m-1, m, m+1, m+2, $u^e=(u_{m-1},\ u_m,\ u_{m+1},\ u_{m+2})$ and $v^e=(v_{m-1},\ v_m, v_{m+1},\ u_{m+2})$ are element parameters where

$$A_{ij}=\int_0^h(i\psi_i\psi_j)d\xi,\ \ B_{ij}=\int_0^h(\psi_i\psi_j^{''})d\xi,\ \ C_{jk}=\int_0^h(\psi_j^{"}\psi_k')d\xi$$

$$D_{ij}=\int_0^h(\psi_i\psi_j)d\xi,\ F_{ijk}=\int_0^h 6(\psi_i\psi_j\psi_k')d\xi,\ G_{ijk}=\int_0^h(\psi_i\psi_j\psi_k)d\xi$$

$$H_{ijk}=\int_0^h 2(\psi_i\psi_j\psi_k')d\xi,\qquad I_{ij}=[(\psi_i\psi_j^{"})]_0^h,\quad J_{ij}=[(\psi_i'\psi_j^{'})]_0^h$$

The element matrices in ((2.17)-(2.18)) are computed as follows:

$$A_{ij}=\frac{ih}{140}\begin{bmatrix}20&129&60&1\\129&1188&933&60\\60&933&1188&129\\1&60&129&20\end{bmatrix}\quad B_{ij}=\frac{3}{10h}\begin{bmatrix}4&-7&2&1\\33&-44&-11&22\\22&-11&-44&33\\1&2&-7&4\end{bmatrix}$$

$$C_{ij}=\frac{3}{2h^2}\begin{bmatrix}-3&-5&7&1\\5&3&-9&1\\-1&9&-3&-5\\-1&-7&5&3\end{bmatrix}\quad D_{ij}=\frac{h}{140}\begin{bmatrix}20&129&60&1\\129&1188&933&60\\60&933&1188&129\\1&60&129&20\end{bmatrix}$$

$$I_{ij}=\frac{6}{h^2}\begin{bmatrix}-1&2&-1&0\\-4&9&-6&1\\-1&6&-9&4\\0&1&-2&1\end{bmatrix}\quad J_{ij}=\frac{9}{h^2}\begin{bmatrix}-1&0&1&0\\0&1&0&-1\\1&0&-1&0\\0&-1&0&1\end{bmatrix}$$

$$G_{ij}(u)=\frac{h}{840}\begin{bmatrix}G_{11}(u)&G_{12}(u)&G_{13}(u)&G_{14}(u)\\G_{21}(u)&G_{22}(u)&G_{23}(u)&G_{24}(u)\\G_{31}(u)&G_{32}(u)&G_{33}(u)&G_{34}(u)\\G_{41}(u)&G_{42}(u)&G_{43}(u)&G_{44}(u)\end{bmatrix}$$

4

$$F_{ij}(v) = \frac{6h}{840}\begin{bmatrix} F_{11}(v) & F_{12}(v) & F_{13}(v) & F_{14}(v) \\ F_{21}(v) & F_{22}(v) & F_{23}(v) & F_{24}(v) \\ F_{31}(v) & F_{32}(v) & F_{33}(v) & F_{34}(v) \\ F_{41}(v) & F_{42}(v) & F_{43}(v) & F_{44}(v) \end{bmatrix};$$

$$H_{ij}(u) = \frac{2h}{840}\begin{bmatrix} H_{11}(u) & H_{12}(u) & H_{13}(u) & H_{14}(u) \\ H_{21}(u) & H_{22}(u) & H_{23}(u) & H_{24}(u) \\ H_{31}(u) & H_{32}(u) & H_{33}(u) & H_{34}(u) \\ H_{41}(u) & H_{42}(u) & H_{43}(u) & H_{44}(u) \end{bmatrix}$$

where
$G_{11}(u)$=(84,463,172,1)(u), $G_{12}(u)$=(463,2889,1275,17)(u),
$G_{13}(u)$=(172,1275,696,17)(u), $G_{14}(u)$=(1,17,17,1)(u)
$G_{21}(u)$=(463,2889,1275,17)(u), $G_{22}(u)$=(2889,23664,15519,696)(u)
$G_{23}(u)$=(1275,15519,15519,1275)(u), $G_{24}(u)$=(17,696,1275,172)(u),
$G_{31}(u)$=(172,1275,696,17)(u), $G_{32}(u)$=(1275,15519,15519,1275)(u),
$G_{33}(u)$=(696,15519,23664,2889)(u), $G_{34}(u)$=(17,1275,2889,463)(u),
$G_{41}(u)$=(1,17,17,1)(u), $G_{42}(u)$=(17,696,1275,172)(u),
$G_{43}(u)$=(17,1275,2889,463)(u), $G_{44}(u)$=(1,172,463,84)(u)

$F_{11}(v)$=(-280,-150,420,10)(v) $F_{12}(v)$=(-1605,-1305,2781,129)(v)
$F_{13}(v)$=(-630,-792,1314,108)(v) $F_{14}(v)$=(-5,-21,21,5)(v)
$F_{21}(v)$=(-1605,-1305,2781,129)(v) $F_{22}(v)$=(-10830,-17640,25002,3468)(v)
$F_{23}(v)$=(-5349,-17541,17541,5349)(v) $F_{24}(v)$=(-108,-1314,792,630)(v)
$F_{31}(v)$=(-630,-792,1314,108)(v) $F_{32}(v)$=(-5349,-17541,17541,5349)(v)
$F_{33}(v)$=(-3468,-25002,17640,10830)(v) $F_{34}(v)$=(-129,-2781,1305,1605)(v)
$F_{41}(v)$=(-5,-21,21,5)(v) $F_{42}(v)$=(-108,-1314,792,630)(v)
$F_{43}(v)$=(-129,-2781,1305,1605)(v) $F_{44}(v)$=(-10,-420,150,280)(v)

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$H_{43}(u)$=(129,2781,1305,1605)(u) $H_{44}(u)$=(-10,-420,150,280)(u)

Here A_{ij}, B_{ij}, C_{jk}, D_{ij}, F_{ijk}, G_{ijk}, H_{ijk}, I_{ij} and J_{ij} are element matrices. so, the new obtained system in matrix form

$$\dot{u} = A^{-1}[\{B - G(u)\}u] \tag{2.19}$$

$$\dot{v} = D^{-1}[H(u)u + (J - I)v - Cv - F(u)v] \tag{2.20}$$

Here $u=(u_{-1},u_0,u_1,...,u_N,u_{N+1})$ and $v=(v_{-1},v_0,v_1,...,v_N,v_{N+1})$ are time dependent constraints, The generalized rows of the combined matrices are:

A = $\frac{ih}{140}$(1,120,1191,2416,1191,120,1)
B = $\frac{3}{10h}$(1,24,15,-80,15,24,1)
C = $\frac{3}{2h^2}$(-1,-8,19,0,-19,8,1)
D = $\frac{h}{140}$(1,120,1191,2416,1191,120,1)
I = $\frac{6}{h^2}$(0,0,0,0,0,0,0)
J = $\frac{9}{h^2}$ (0,0,0,0,0,0,0)
G(u) = $\frac{h}{840}${(1,17,17,1,0,0,0)u,(17,868,2550,868,17,0,0)u,(17,2550,18871,18871,2550,17,0)u, (1,868,18871,47496,18871,868,1)u,(0,17,2550,18871,18871,2550,17)u,(0,0,17,868,2550,868, 17)u,(0,0,0,1,17,17,1)u}

F(v) = $\frac{6h}{840}$ {(-5,-21,21,5,0,0,0)v,(-108,-1944,0,1944,108,0,0)v,(-129,-8130,-17841,17841,8130, 129,0)v, (-10,-3888,-35682,0,35682,3888,10)v,(0,-129,-8130,-17841,17841,8130,129)v,(0,0,- 108,-1944,0,1944,108)v,
(0,0,0,-5,-21,21,5)v}

H(u) = $\frac{2h}{840}$ {(-5,-21,21,5,0,0,0)u,(-108,-1944,0,1944,108,0,0)u,(-129,-8130,-17841,17841,8130, 129,0)u, (-10,-3888,-35682,0,35682,3888,10)u,(0,-129,-8130,-17841,17841,8130,129)u,(0,0,- 108,-1944,0,1944,108)u, (0,0,0,-5,-21,21,5)u }

The system equations (2.19) and (2.20) has $(N+3)\times(N+1)$ ordered unknown equations. if we use time dependent boundary condition in Eqs.(2.13) and (2.14) with $m=0$, then so parameters can be written as other parameters;

$$u_{-1},\ v_{-1} \rightarrow u_0,\ u_1 \text{ and } v_0, v_1 \text{ ; when we take } m=0$$

Similarly

$$u_{N+1}\ ,\ v_{N+1} \rightarrow u_{N-1}, u_N \text{ and } v_{N-1}, v_N \text{ we take } m=N$$

Then, the system of Eqs. (2.19) and(2.20) will be two matrix systems of $(N+1)\times(N+1)$ orders.These equations of systems will be solved by RK^4 (Runge-Kutta fourth order method) to known initial condition u_j^0 and v_j^0 with nodal points x_m for m=0(1)N as follows:

$$u(x_m,0)=\theta_N(x_m,0)$$

$$v(x_m,0)=\upsilon_N(x_m,0)$$

If we write the system explicitly as

$$\theta_N(x_0,0)=u_{-1}+4u_0+u_1=u(x_0,0),$$

$$\theta_N(x_1,0)=u_0+4u_1+u_2=u(x_1,0),$$

$$\theta_N(x_2,0)=u_1+4u_2+u_3=u(x_2,0),$$

6

$$\vdots$$

$$\theta_N(x_N,0) = u_{N-1} + 4u_N + u_{N+1} = u(x_N,0),$$

and

$$\upsilon_N(x_0,0) = \upsilon_{-1} + 4\upsilon_0 + \upsilon_1 = \upsilon(x_0,0),$$
$$\upsilon_N(x_1,0) = \upsilon_0 + 4\upsilon_1 + \upsilon_2 = \upsilon(x_1,0),$$
$$\upsilon_N(x_2,0) = \upsilon_1 + 4\upsilon_2 + \upsilon_3 = \upsilon(x_2,0),$$
$$\vdots$$
$$\upsilon_N(x_N,0) = \upsilon_{N-11} + 4\upsilon_N + \upsilon_{N+1} = \upsilon(x_N,0),$$

if we write u_{-1} , $u_{N+1} \to u_0$, u_N, and υ_{-1} , $\upsilon_{N+1}\to$ υ_0 and υ_N respectively. then we get a new system $(N+1)\times(N+1)$ order in matrix form as :

$$\begin{bmatrix} 4 & 2 & & & & & & \\ 1 & 4 & 1 & & & & & \\ & 1 & 4 & 1 & & & & \\ & & & . & & & & \\ & & & & . & & & \\ & & & & & . & & \\ & & & & & 1 & 4 & 1 \\ & & & & & & 2 & 4 \end{bmatrix} \begin{bmatrix} u_0 \\ u_1 \\ u_2 \\ . \\ . \\ . \\ u_{N-1} \\ u_N \end{bmatrix} = \begin{bmatrix} u(x_0,0) \\ u(x_1,0) \\ u(x_2,0) \\ . \\ . \\ . \\ u(x_{N-1},0) \\ u(x_N,0) \end{bmatrix} \tag{2.21}$$

and

$$\begin{bmatrix} 4 & 2 & & & & & & \\ 1 & 4 & 1 & & & & & \\ & 1 & 4 & 1 & & & & \\ & & & . & & & & \\ & & & & . & & & \\ & & & & & . & & \\ & & & & & 1 & 4 & 1 \\ & & & & & & 2 & 4 \end{bmatrix} \begin{bmatrix} \upsilon_0 \\ \upsilon_1 \\ \upsilon_2 \\ . \\ . \\ . \\ \upsilon_{N-1} \\ \upsilon_N \end{bmatrix} = \begin{bmatrix} \upsilon(x_0,0) \\ \upsilon(x_1,0) \\ \upsilon(x_2,0) \\ \upsilon(x_{N-1},0) \\ \upsilon(x_N,0) \end{bmatrix} \tag{2.22}$$

By Matleb solving the algebraic Equations (2.21) and (2.22) with initial parameters u_j^0 and υ_j^0 are gained for j=0(1)N.

3 Numerical Scheme

Non-Linear waves propagations and interaction are investigated to the system of equations (2.1)-(2.2) numerically for numerous values of x and t. L_2 , L_∞ and L'_2 , L'_∞ are error norms and used to investigate consistency with numerical solutions(Soliton) for $\theta(x,t)$ and $\upsilon(x,t)$ respectively for initial conditions for the Sch-KdV equation.

$$\theta(x,0) = f(x) = 9\sqrt{2}\, e^{i\alpha x}\, k^2 \mathrm{sech}^2(kx), \tag{3.1}$$

$$v(x,0) = g(x) = \frac{\alpha + 16k^2}{3} - 6k^2 \tanh^2(kx) \tag{3.2}$$

$$L_2 = \|\theta - \theta_N\|_2 = \sqrt{h \sum_{j=-1}^{N+1} \left|\theta_j - (\theta_N)_j\right|^2} \tag{3.3}$$

$$L_\infty = \|\theta - \theta_N\|_\infty = \underset{0 \le j \ge N}{Max} \left|\theta_j - (\theta_N)_j\right| \tag{3.4}$$

And

$$L'_2 = \|v - v_N\|_2 = \sqrt{h \sum_{j=-1}^{N+1} \left|v_j - (v_N)_j\right|^2} \tag{3.5}$$

$$L'_\infty = \|v - v_N\|_\infty = \underset{0 \le j \ge N}{Max} \left|v_j - (v_N)_j\right| \tag{3.6}$$

Numerical error L_2 *and* L_∞ For $\theta(x,t)$ with $k = \sqrt{2},\ \alpha = 1/20$

h	$\Delta t = 0.001$	$\Delta t = 0.002$	$\Delta t = 0.003$	$\Delta t = 0.01$
	L_2 ; L_∞	L_2 ; L_∞	L_2 ; L_∞	L_2 ; L_∞
0.2	15.82×10^{-7} ; 51.24×10^{-7}	30.71×10^{-7} ; 95.17×10^{-7}	41.39×10^{-7} ; 97.41×10^{-7}	55.56×10^{-7} ; 99.56×10^{-7}
0.4	54.21×10^{-7} ; 55.88×10^{-7}	63.56×10^{-7} ; 68.67×10^{-7}	71.39×10^{-7} ; 70.70×10^{-7}	86.66×10^{-7} ; 85.01×10^{-7}
0.625	57.24×10^{-7} ; 59.82×10^{-7}	68.21×10^{-7} ; 61.23×10^{-7}	78.29×10^{-7} ; 68.21×10^{-7}	87.21×10^{-7} ; 82.01×10^{-7}
0.8	68.19×10^{-7} ; 64.21×10^{-7}	72.21×10^{-7} ; 59.52×10^{-7}	80.19×10^{-7} ; 63.11×10^{-7}	89.21×10^{-7} ; 78.21×10^{-7}
0.1	75.21×10^{-7} ; 72.24×10^{-7}	75.11×10^{-7} ; 52.11×10^{-7}	83.12×10^{-7} ; 59.29×10^{-7}	91.11×10^{-7} ; 72.31×10^{-7}

Numerical error L_2 *and* L_∞ For $v(x,t)$

h	$\Delta t = 0.001$	$\Delta t = 0.002$	$\Delta t = 0.003$	$\Delta t = 0.01$
	L'_2 ; L'_∞	L'_2 ; L'_∞	L'_2 ; L'_∞	L'_2 ; L'_∞
0.25	07.85×10^{-8} ; 05.68×10^{-8}	08.75×10^{-7} ; 07.05×10^{-7}	15.16×10^{-7} ; 06.65×10^{-7}	19.56×10^{-7} ; 08.75×10^{-7}
0.5	09.95×10^{-8} ; 06.95×10^{-8}	10.72×10^{-7} ; 08.75×10^{-7}	20.61×10^{-7} ; 08.85×10^{-7}	25.11×10^{-7} ; 09.85×10^{-7}
0.625	10.01×10^{-8} ; 07.02×10^{-8}	13.56×10^{-7} ; 09.11×10^{-7}	22.56×10^{-7} ; 09.21×10^{-7}	26.92×10^{-7} ; 10.96×10^{-7}
0.8	12.21×10^{-8} ; 08.11×10^{-8}	15.11×10^{-7} ; 10.21×10^{-7}	24.11×10^{-7} ; 10.09×10^{-7}	28.11×10^{-7} ; 12.11×10^{-7}
0.1	14.02×10^{-8} ; 09.75×10^{-8}	19.21×10^{-7} ; 11.25×10^{-7}	27.72×10^{-7} ; 12.21×10^{-7}	31.27×10^{-7} ; 14.25×10^{-7}

In figure 1 and 2 nonlinear wave propagation and its travelling wave solution is presented. The coupled equations (2.1) and (2.2) are plotted for some fix values of k, α,h and t ($-5 < t < 5$). the space step is taken as 0.001. It is shown in the figure that the solution of said equation exhibit a soliton for the small values of x ($0 \leq x \leq 0.1$). If we extend the range of x ($-15 \leq x \leq 15$) the solution converted from soliton to a wave natured system. A solitary wave interaction is presented in the figure 3 for the same values of k, α, h and step lengths with

8

Solitary wave propagation for model equations

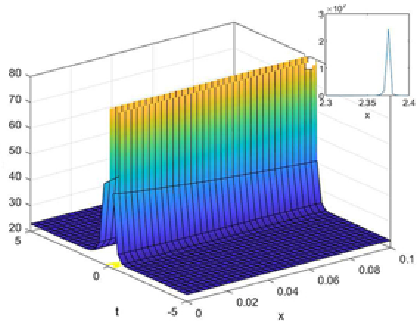

Figure 1. Modulus in 3D, 2D plot the *solitary wave propagation of θ when* $k=\sqrt{2},\ \alpha=1/20, h=0.4$ $\Delta t=0.001,\ \Delta x=0.001, -5\le t\le 5,\ 0\le x\le 0.1,$

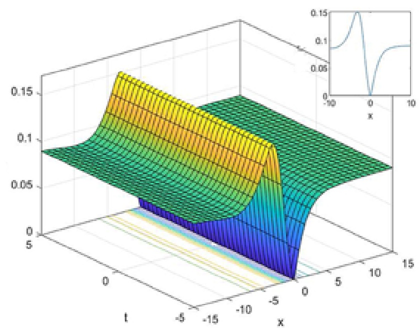

Figure 2. Modulus in 3D, 2D plot the *solitary wave propagation of υ when* $k=\sqrt{2},\ \alpha=1/20, h=0.4$ $\Delta t=0.001,\ \Delta x=0.001, -5\le t\le 5,\ -15\le x\le 15,$

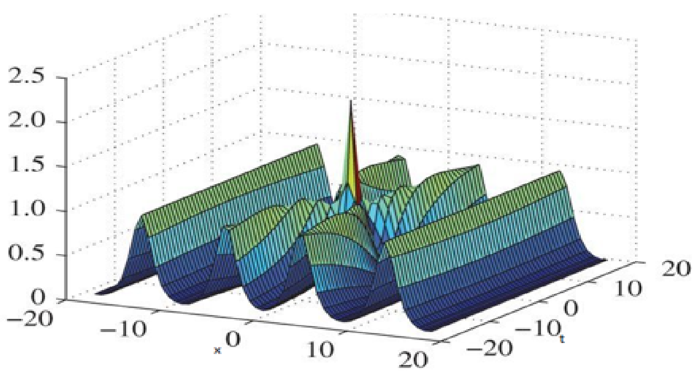

Fig.-3 Solitary wave interaction
$k=\sqrt{2},\ \alpha=1/20,\ \Delta t=0.02,\ \Delta x=0.02,\ -20\le(x,t)\le 20,$

large values of x and t $(-20 \le (x,t) \le 20)$. It clearly exhibit that solitons are developed when the values of x and t coincides. For different values of x and t the system represent the travelling wave solution.

4 Conclusions

In the present paper, we have investigated numerically a physical model for wave propagation in a nonlinear, dispersive medium i.e a relativistic plasma. A Galerkin finite element Scheme is exhibited to locate Solitary wave(Solitons) propagation and interactions in plasma for Schrödinger - KortewegDe Vries (Sch-KdV) equations. The new obtained systems (finite element formulation) solved

by RK^4 (Runge-Kutta fourth order method). The different values of x, t and error norms L_2 , L_∞ are used for numerical solutions of Sch-KdV equations.The numerical results obtained by this method are in good agreement with the exact solutions available in the literature. The errors obtained by the proposed method are less when compared with those of available in the literature. The solitary wave solution in fig.-1, 2 and its interaction in fig.-3 of this system are presented which are new. here, we learn that this method will emulates development of many exact travelling wave solutions with new solitons.This scheme is a significant instrument for Non-linear evolution equations (NLEEs).
The advantages of the present scheme for oscillatory problems are discussed in detail. It can be expected that the main ideas will also be useful for other physical problems being highly oscillatory in nature, e.g., the nonlinearized model.

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